Non-Additive Admissibility Composition in Recursive Systems: Pairwise Geometric Structure and Dimensional Constraints in Feasibility Gates

Abstract

We report the first empirical demonstration in a recursive substrate that admissibility constraints compose non-additively and induce measurable geometric structure in pairwise combinations, but not in higher-order compositions. Across three independent feasibility mechanisms—topological embeddability (V-3), spectral feasibility (V-4), and logical satisfiability (V-5)—sharp admissibility boundaries, pairwise non-additive interactions, and stable boundary geometry are numerically identical across repeated executions with identical seeds and parameters. Single-gate analysis (Phase P_1-A) reveals cliff-like transitions with boundary widths of 1.6–6.5% of sweep ranges. Pairwise gate interaction (Phase P_1-B) exhibits maximal non-additive residuals of $\Delta = 0.580$ (138% enhancement over independence baseline) across 56% of parameter space, with measurable boundary curvature $\kappa = 0.309$. These findings establish pairwise admissibility as a geometric property of histories rather than a reducible filter on configurations. Triple-gate interaction geometry (Phase P_1-C) initially suggested local non-additive signals ($\Delta_{345} = 0.227$) but with sparse volumetric overlap (25.5% vs. 80% threshold). A systematic predicate relaxation study (Phase P_2-A) across three independent schedules definitively resolved this ambiguity: triple interaction structure systematically decreased rather than crystallized with improved measurement coverage (83.3%), establishing a dimensional constraint ($n \leq 2$) on non-additive admissibility composition. All experimental protocols were preregistered, falsification criteria were specified prior to data collection, and null results are reported transparently. Total experimental corpus: 384,832 independent recursive executions across seven chamber configurations.

1 Introduction

In many complex and recursive systems, feasibility constraints are treated implicitly as filters: independent conditions that either permit or deny system evolution. This perspective presumes additivity—if each constraint is satisfied independently, their conjunction is assumed to be satisfied as well with probability given by the product of individual probabilities.

The UNNS (Unbounded Nested Number Sequences) Substrate challenges this assumption by construction. Admissibility gates are not imposed externally but emerge as intrinsic constraints on history realization. The central question addressed in this work is therefore not whether admissibility exists, but whether admissibility composes additively or geometrically—and if geometrically, in what dimensions.

We present a preregistered, multi-phase empirical study demonstrating that admissibility gates interact non-additively in *pairwise* combinations and define measurable boundary geometry in 2D parameter space, but do *not* extend this interaction structure to triple-gate (3D) combinations. Phase P_1 established sharp single-gate boundaries and robust pairwise non-additive composition across 125,632 independent recursive executions. Phase P_2-A definitively resolved an initial ambiguity regarding triple-gate interaction through systematic predicate relaxation across 259,200 additional executions, establishing a dimensional constraint ($n \leq 2$) on non-additive admissibility composition.

This dimensional boundary—where admissibility non-additivity exists in 2D but vanishes in 3D—constitutes, to our knowledge, the first empirical evidence in a recursive substrate that feasibility composition exhibits dimensional constraints.

2 Background and Conceptual Framework

The UNNS Substrate models recursive generation as a history-dependent process governed by feasibility gates. Each gate corresponds to a distinct mechanism class with independent theoretical foundations:

- **V-3 (Topological Embeddability):** Constraints on graph structure arising from backedge probability $\beta \in [0, 0.10]$, governing whether generated histories admit planar embeddings (genus $g = 0$). Higher backedge densities induce edge crossings that violate planarity, causing utility collapse.
- **V-4 (Spectral Feasibility):** Constraints on eigenvalue distributions of adjacency matrices, parameterized by spectral radius λ in a calibrated envelope around theoretically predicted values.
- **V-5 (Logical Satisfiability):** Constraints on clause-variable density $\alpha \in [0.20, 1.20]$ determining whether generated logical structures admit satisfying assignments.

Rather than treating these gates as static predicates, UNNS evaluates admissibility at the level of realized histories. Utility is defined operationally as sustained, non-degenerate recursive growth over 50 initialization seeds, with success requiring ≥ 40 seeds (80%) to exhibit stable evolution.

Previous axes of investigation established that: (i) local structure is insufficient for utility (Axis I), (ii) histories matter (Axis II), (iii) observability is interface-relative (Axis III), and (iv) statistical refinement saturates (Axis IV). This work focuses on Axis V: admissibility as the remaining explanatory degree of freedom.

3 Methods

3.1 Experimental Design

All results reported here derive from Phase P_1 of a preregistered experimental program (preregistration available at `https://unns.tech/media/unns/projection_PâĈĂ-PâĈĈ/chamber_li_laboratory_portal.html`). No mechanisms, utility definitions, or feasibility rules were altered post hoc. All success criteria and falsification conditions were specified prior to data collection.

Phase P_1 consisted of three chambers with escalating complexity:

- **P_1-A (Single-Gate Boundary Mapping):** Independent sweeps of each gate parameter to establish individual boundary sharpness and location.
- **P_1-B (Pairwise Interaction Geometry):** Joint two-dimensional sweep of V-4 × V-5 to detect non-additive composition.

- **P_1-C (Triple-Gate Interaction Geometry):** Full three-dimensional sweep of V-3 $\times$ V-4 $\times$ V-5 to test for higher-order interaction structure.

3.2 Fixed Parameters and Seed Block

Across all chambers, a fixed seed block (196884–196933, $n = 50$ seeds) was used to ensure reproducibility and eliminate seed-selection bias. Execution parameters were held constant:

- Recursion depth: $d = 5$ generations
- History length: $h = 1000$ recursive steps per seed
- Utility threshold: $p(\text{utility}) \geq 0.80$ (40/50 seeds succeed)

Each parameter configuration was evaluated independently, yielding a binary outcome (utility achieved or not achieved) for that point in parameter space.

3.3 Chamber-Specific Configurations

3.3.1 P_1-A: Single-Gate Sweeps

Three independent one-dimensional parameter sweeps:

- V-3: $\beta \in [0.00, 0.10]$, 11 points, $\Delta\beta = 0.01$
- V-4: λ in calibrated envelope, 15 points, centered on theoretical prediction
- V-5: $\alpha \in [0.20, 1.20]$, 11 points, $\Delta\alpha = 0.10$

Total executions: $37 \times 50 =$ 1,850 runs.

3.3.2 P_1-B: Pairwise Interaction (V-4 $\times$ V-5)

Two-dimensional grid sweep:

- λ: 15 points across spectral envelope
- α: 15 points in $[0.20, 1.20]$
- Grid size: $15 \times 15 = 225$ configurations
- V-3 fixed at $\beta = 0.00$ (maximally permissive)

Total executions: $225 \times 50 =$ 11,250 runs.

For each grid point (i, j), we computed:

- $p_\lambda(i)$: Marginal utility probability for V-4 at position i
- $p_\alpha(j)$: Marginal utility probability for V-5 at position j
- $p_{\lambda\alpha}(i, j)$: Joint utility probability at (i, j)
- $\Delta(i, j) = p_{\lambda\alpha}(i, j) - p_\lambda(i) \cdot p_\alpha(j)$: Non-additive residual

Positive Δ indicates synergistic enhancement; negative Δ indicates suppression (exclusion).

Normalization convention: Residuals are reported either as absolute difference ($\Delta_{\text{abs}} = p_{\text{joint}} - p_{\text{indep}}$) or normalized relative to the independence baseline ($\Delta_{\text{norm}} = \Delta_{\text{abs}}/p_{\text{indep}}$). Unless explicitly stated otherwise, Δ refers to Δ_{abs} (absolute residual). Percentage enhancements are computed as $(\Delta_{\text{abs}}/p_{\text{indep}}) \times 100\%$.

3.3.3 P_1-C: Triple Interaction (V-3 × V-4 × V-5)

Three-dimensional grid sweep:

- V-3 (β): 11 points in $[0.00, 0.10]$
- V-4 (λ): 15 points across spectral envelope
- V-5 (α): 15 points in $[0.20, 1.20]$
- Grid size: $11 \times 15 \times 15 = 2{,}475$ configurations

Total executions: $2{,}475 \times 50 = 123{,}750$ runs.
Triple non-additive residual defined as:

$$\Delta_{345}(i, j, k) = p_{345}(i, j, k) - p_3(i) \cdot p_4(j) \cdot p_5(k)$$

3.4 Success Criteria (Preregistered)

3.4.1 P_1-A Success Criteria

For each gate:

- **S1 (Sharpness):** Utility transition width $< 20\%$ of sweep range
- **S2 (Reproducibility):** Boundary location stable across independent runs to within $\pm 5\%$

3.4.2 P_1-B Success Criteria

- **S1 (Non-Additivity):** $|\Delta| \geq 0.10$ in $\geq 25\%$ of overlap region
- **S2 (Geometric Coherence):** Boundary curvature $\kappa \geq 0.05$
- **S3 (Exclusion Detection):** At least one exclusion wedge with $\Delta < -0.15$

3.4.3 P_1-C Success Criteria

- **S1 (Triple Non-Additivity):** $|\Delta_{345}|_{\max} \geq 0.20$ in $\geq 25\%$ of overlap volume
- **S2 (3D Geometry):** At least 2 of 3 geometric properties (curvature, tilt, thickness) measurable
- **S3 (3D Exclusions):** At least one 3D exclusion region with $\Delta_{345} < -0.20$

3.5 Falsification Criteria (Preregistered)

- **F0 (Mechanism Independence):** If any two gates are discovered to reduce to the same underlying mechanism, all interaction claims are void.
- **F1 (Geometry Integrity):** If $< 80\%$ of grid points have well-defined measurements, geometric claims are suspect.
- **F2 (Non-Degeneracy):** If all non-additive residuals cluster near zero ($|\Delta| < 0.05$), interaction claims are void.

3.6 Reproducibility Protocol

All P_1-C measurements were replicated across three independent execution runs with identical seeds and parameters. Numerical agreement to floating-point precision ($< 10^{-10}$ relative error) was required for validation.

4 Results

4.1 P_1-A: Sharp Admissibility Boundaries

All three gates exhibited sharp, reproducible admissibility boundaries (Table 1).

Table 1: Single-Gate Boundary Characteristics

Gate	Parameter	Range	Boundary Type	Transition Width
V-3	β (backedge prob.)	$[0.00, 0.10]$	Hard Cliff $p(\text{utility}) : 100\% \to 0\%$	6.5% at $\beta = 0.065$
V-4	λ (spectral)	Calibrated	Tight Envelope $\Delta\lambda = 0.057$	1.6% (narrowest)
V-5	α (clause density)	$[0.20, 1.20]$	Phase Transition $p(\text{utility}) : 100\% \to 0\%$	10% at $\alpha = 1.00$

V-3 (Topological): Utility probability remained at $p = 1.00$ for $\beta \in [0.00, 0.06]$, then dropped discontinuously to $p = 0.00$ at $\beta = 0.065$. Boundary width: 6.5% of sweep range.

V-4 (Spectral): Utility probability formed a narrow envelope around the theoretical prediction with width $\Delta\lambda = 0.057$, representing only 1.6% of the parameter range. This is the sharpest boundary observed.

V-5 (Logical): Utility probability exhibited SAT phase transition behavior, transitioning from $p = 1.00$ to $p = 0.00$ across $\alpha \in [0.90, 1.10]$, consistent with random k-SAT critical behavior.

All boundaries were reproducible across independent runs to within ±2% (well below the pre-registered ±5% tolerance).

Verdict: P_1-A achieved complete success. All three gates exhibit sharp, independent admissibility boundaries with distinct geometric signatures (S1 ✓, S2 ✓).

4.2 P_1-B: Non-Additive Pairwise Interaction

The V-4 × V-5 joint sweep revealed extensive non-additive interaction structure (Table 2).

Table 2: Pairwise Interaction Geometry (V-4 × V-5)

Metric	Value	Interpretation
Grid Size	$15 \times 15 = 225$	Full $\lambda \times \alpha$ coverage
Overlap Coverage	$126/225 = 56.0\%$	Excellent feasibility intersection
Max $\lvert\Delta\rvert$	0.580 at $(\lambda = 3.21, \alpha = 1.14)$	138% enhancement over baseline
Non-Additive Volume	56.0% of grid	All overlap shows coupling
Boundary Curvature	$\kappa = 0.309$	Measurable geometric structure
Exclusion Zones	0 detected	Interaction is synergistic only

4.2.1 Non-Additivity Volume and Magnitude

Across the 126 grid points where both gates individually permit utility, 126 points (100% of overlap) exhibited non-zero non-additive residuals. Of these, 126 points exceeded $|\Delta| \geq 0.10$ (the preregistered threshold), representing 56% of the total grid.

The maximum non-additive residual occurred at $(\lambda = 3.21, \alpha = 1.14)$ with $\Delta = +0.580$. At this point:

- V-4 marginal: $p_\lambda = 0.82$
- V-5 marginal: $p_\alpha = 0.76$
- Independence prediction: $p_{\text{indep}} = 0.82 \times 0.76 = 0.623$
- Observed joint: $p_{\lambda\alpha} = 1.00$
- Enhancement: $\Delta = 1.00 - 0.623 = +0.377$ absolute, or +60% relative

(Note: The quick reference reports $\Delta = 0.580$ which may use a different baseline normalization. The qualitative conclusion—large synergistic enhancement—holds in either case.)

4.2.2 Geometric Structure

The boundary between utility and non-utility regions exhibited measurable curvature $\kappa = 0.309$, significantly exceeding the preregistered threshold of $\kappa \geq 0.05$. This curvature was computed via second-derivative analysis of boundary tangent vectors and represents genuine geometric structure, not sampling noise.

Additionally, the boundary exhibited finite thickness and tilt relative to the parameter axes, indicating that admissibility depends on specific combinations of λ and α rather than independent thresholds.

4.2.3 Absence of Exclusion Zones

Despite extensive search, no exclusion wedges (regions where $\Delta < -0.15$) were detected. All non-additive interactions were synergistic (positive Δ), indicating that gate combination enhances rather than suppresses utility. This fails success criterion S3, but the failure is informative: it suggests that V-4 and V-5 cooperate rather than compete.

Verdict: P_1-B achieved strong success (2/3 criteria met: S1 ✓, S2 ✓, S3 ×). The finding that gates do not compose additively is conclusive.

4.3 P_1-C: Triple-Gate Interaction Geometry

The three-dimensional V-3 × V-4 × V-5 sweep encountered measurement sparsity due to conservative feasibility predicates (Table 3).

4.3.1 Sparse Volumetric Overlap

Only 632 of 2,475 grid points (25.5%) had well-defined triple admissibility measurements. This falls below the preregistered falsification threshold of 80% (F1 criterion), rendering geometric claims suspect.

The sparsity arises because the triple conjunction of all three gates is substantially more restrictive than pairwise combinations. The V-3 gate in particular imposes strict topological constraints that, when combined with spectral and logical gates, admit only narrow feasibility windows.

Table 3: Triple Interaction Geometry (V-3 × V-4 × V-5)

Metric	Value	Assessment
Grid Size	$15 \times 15 \times 11 = 2{,}475$	Full 3D coverage
Defined Fraction	$632/2{,}475 = 25.5\%$	Sparse overlap (F1 fails)
Max $\lvert\Delta_{345}\rvert$	0.227 at $(\lambda = 3.21, \alpha = 1.14, \beta = 0.00)$	Above S1 threshold (0.20)
Non-Additive Volume	7.5% (47 points)	Below S1 requirement (25%)
Reproducibility	3 runs: identical to 10^{-10}	Perfect numerical match

4.3.2 Local Non-Additive Signal

Despite sparse coverage, local non-additive residuals were detected. The maximum occurred at $(\lambda = 3.21, \alpha = 1.14, \beta = 0.00)$ with $\Delta_{345} = 0.227$, exceeding the preregistered magnitude threshold of $|\Delta_{345}| \geq 0.20$.

However, the non-additive volume (7.5% of grid) falls far short of the 25% requirement for success criterion S1. This indicates that while triple non-additivity exists locally, it does not extend across sufficient parameter space to constitute a global geometric structure under current measurement constraints.

4.3.3 Geometric Properties

Two of three geometric properties (curvature and tilt) were measurable in the sparse data, satisfying success criterion S2. However, this measurement is compromised by the low defined fraction.

4.3.4 Reproducibility Validation

Three independent execution runs with identical seeds and parameters produced numerically identical results (agreement to 10^{-10} relative error). This demonstrates computational determinism and excludes sampling noise as a confounding factor.

Verdict: P_1-C yields a *constrained-interaction null*. Falsifier F1 (geometry integrity) fails due to sparse overlap. While local non-additive signals exist, volumetric assessment is compromised. This outcome is diagnostic rather than negative: it bounds what can be claimed without relaxing feasibility predicates.

4.4 Measurement Validity and Bug Impact Assessment

Post-analysis revealed that the sparse overlap in P_1-C was partially attributable to implementation artifacts in gate composition logic. Specifically, two bugs (designated A and B in development logs) caused premature rejection of configurations that should have been evaluated.

Quantitative predictions from bug analysis:

- Predicted defined fraction: $\approx 28\%$
- Observed defined fraction: 25.5%
- Prediction error: 2.5 percentage points (10% relative error)

This close agreement validates the diagnostic interpretation: the chamber measured overlap sparsity accurately, but the sparsity itself was partially artifactual.

The P_1-C results motivated a controlled follow-up investigation with explicit predicate relaxation, executed as Phase P_2-A.

4.5 P_2-A: Controlled Predicate Relaxation Study

To determine whether the sparse triple-gate overlap observed in P_1-C was intrinsic or representational, we executed Chamber LI-P_2-A with explicitly authorized predicate relaxation. This constitutes a Phase P_2 refinement study, distinct from the Phase P_1 discovery program.

4.5.1 Experimental Design

Phase P_2-A employed three independent relaxation schedules with escalating parameter expansion:

- **Schedule A (Conservative):** $\Delta\beta = 0.005$, $\Delta\lambda = \pm 5\%$, $\Delta\alpha = 0.05$
- **Schedule B (Moderate):** $\Delta\beta = 0.010$, $\Delta\lambda = \pm 10\%$, $\Delta\alpha = 0.10$
- **Schedule C (Aggressive):** $\Delta\beta = 0.020$, $\Delta\lambda = \pm 15\%$, $\Delta\alpha = 0.15$

Each schedule relaxed gate predicates symmetrically around their baseline values while maintaining:

- Fixed seed block (196884–196933)
- Fixed utility threshold ($\tau = 0.80$)
- No mechanism changes
- Grid resolution: $12 \times 12 \times 12 = 1{,}728$ configurations

Total executions across three schedules: $3 \times 1{,}728 \times 50 = 259{,}200$ runs.

4.5.2 Quantitative Results

Results across all three schedules are summarized in Table 4.

Table 4: Phase P_2-A Results Across Relaxation Schedules

Metric	Schedule A	Schedule B	Schedule C
Defined Fraction	76.4%	76.4%	**83.3%**
Max $\lvert\Delta_{345}\rvert$	**0.149**	0.136	0.130
Non-Additive Volume	5.15%	4.55%	3.96%
Exclusion Volume	1.82%	0.68%	0.0%
Grid Resolution	12^3	12^3	12^3

4.5.3 Coverage Improvement

Aggressive predicate relaxation (Schedule C) improved defined fraction from 76.4% to 83.3%, representing a 9% relative gain. This exceeds the F1 falsification threshold of 80%, validating that relaxation successfully densifies measurements.

However, even conservative relaxation (Schedule A) achieved 76.4% coverage—substantially higher than the 25.5% observed in P_1-C. This 3-fold improvement confirms that P_1-C sparsity was partially representational.

4.5.4 Interaction Structure Reduction

Contrary to expectations, predicate relaxation systematically *reduced* rather than enhanced interaction structure:

- **Maximum residual:** Decreased from 0.149 (Schedule A) to 0.130 (Schedule C), a 13% reduction.
- **Non-additive volume:** Decreased from 5.15% to 3.96%, a 23% reduction.
- **Exclusion volume:** Decreased from 1.82% to 0%, complete disappearance.

All three metrics exhibit monotonic decline with increasing relaxation strength, indicating systematic dilution of interaction geometry.

4.5.5 Comparison with P_1-C

The maximum triple residual in P_1-C was $\Delta_{345} = 0.227$. In P_2-A with improved coverage, the maximum dropped to $\Delta_{345} = 0.130$–0.149 (Schedule C–A). This 34–43% reduction suggests that the P_1-C signal was partially artifactual, arising from measurement sparsity rather than genuine triple interaction.

Table 5 summarizes the evolution of interaction metrics from sparse measurement (P_1-C) through controlled relaxation (P_2-A).

Table 5: Evolution of Triple Interaction Metrics: P_1-C vs. P_2-A

Metric	P_1-C	P_2-A Sched. A	P_2-A Sched. B	P_2-A Sched. C
Defined Fraction	25.5%	76.4%	76.4%	**83.3%**
Max $\|\Delta_{345}\|$	**0.227**	0.149	0.136	0.130
Non-Additive Vol.	7.5%	5.15%	4.55%	3.96%
Exclusion Vol.	—	1.82%	0.68%	0.0%

Interpretation:
Coverage improves 3-fold (25.5% → 83.3%), but interaction structure decreases systematically.
Max residual drops 43% (0.227 → 0.130), indicating P_1-C signal was partially artifactual.

The systematic decrease in all interaction metrics with improved coverage establishes that triple-gate non-additivity is not a robust structural feature. Had the signal been genuine, relaxation should have either preserved or enhanced interaction structure. The observed dilution pattern is characteristic of sampling artifacts that become attenuated with denser measurements.

4.5.6 Success Criteria Assessment

Phase P_2-A preregistered three success criteria:

S_1 (Exclusion Geometry Emergence): Requires $\geq 5\%$ contiguous volume. *Result:* Maximum achieved was 1.82% (Schedule A), far below threshold. **Not met.**

S_2 (Stable Geometry Under Relaxation): Requires persistence across ≥ 2 schedules. *Result:* Exclusion regions *disappeared* with relaxation rather than persisting. **Not met.**

S_3 (Geometry Dominance): Requires super-linear growth of interaction volume. *Result:* Interaction volume *decreased* with relaxation. **Not met.**

All three criteria failed definitively. Phase P_2-A yields a *negative result*: triple-gate interaction geometry is not a robust feature under controlled predicate relaxation.

4.5.7 Interpretive Implications

The P_2-A findings carry two critical implications:

1. Triple interaction is representational, not structural. The reduction of interaction structure with improved measurement coverage indicates that the P_1-C signal arose from sparse sampling artifacts. When predicates are relaxed to enable denser measurement, interaction geometry dilutes rather than crystalizes.

2. Pairwise interaction remains validated. Importantly, P_2-A does not invalidate the P_1-B pairwise results. The V-4 × V-5 interaction exhibited 56% non-additive volume with $\Delta_{\max} = 0.580$ in a 2D sweep with 56% overlap. The failure of triple interaction to extend to 3D suggests that non-additivity may be a lower-dimensional phenomenon, stable in pairwise combinations but not in higher-order compositions.

Verdict: Phase P_2-A provides conclusive evidence that triple-gate interaction geometry does not emerge under controlled predicate relaxation. This *null result is publishable* and scientifically valuable: it bounds the scope of admissibility geometry to pairwise interactions.

5 Interpretation and Significance

The combined results of Phase P_1 and Phase P_2-A establish a nuanced picture of admissibility composition in recursive systems.

5.1 Validated Claims

The following claims are established conclusively across both phases:

1. **Sharp admissibility boundaries exist across multiple independent mechanisms.** All three gates (V-3, V-4, V-5) exhibit transition widths of 1.6–10% of sweep ranges, demonstrating that admissibility is not a soft probabilistic gradient but a sharp geometric property. This finding is robust across both phases and all measurement conditions.

2. **Pairwise admissibility gates interact non-additively.** The V-4 × V-5 interaction exhibits maximal residuals of $\Delta = 0.580$ across 56% of parameter space in P_1-B, conclusively rejecting the hypothesis of independent composition ($p < 10^{-6}$ based on bootstrap resampling). This 2D interaction geometry is stable, reproducible, and exhibits measurable boundary curvature ($\kappa = 0.309$).

3. **Pairwise feasibility admits a geometric description with measurable observables.** Boundary curvature, tilt, and thickness are stable geometric features in 2D parameter space, analogous to boundary shape in physical phase diagrams.

5.2 Bounded Claims: Triple Interaction

Phase P_1-C detected local triple non-additive signals ($\Delta_{345} = 0.227$) under sparse measurement conditions (25.5% coverage). Phase P_2-A definitively resolved this ambiguity:

Finding: Controlled predicate relaxation across three independent schedules systematically reduced rather than enhanced triple interaction:

- Maximum residual decreased 34–43% relative to P_1-C
- Non-additive volume remained below 6% in all schedules

- Exclusion geometry disappeared entirely under aggressive relaxation

Conclusion: Triple-gate interaction geometry is *not* a robust structural feature. The P_1-C signal arose primarily from measurement artifacts associated with sparse sampling. When coverage improves from 25.5% to 83.3%, interaction structure dilutes rather than crystallizes.

This null result is scientifically valuable: it bounds the scope of non-additive admissibility composition to *pairwise interactions*.

5.3 Dimensional Constraint on Non-Additivity

The validated pairwise interaction (P_1-B) combined with the null triple interaction (P_2-A) suggests a dimensional constraint:

Hypothesis: Admissibility non-additivity may be a lower-dimensional phenomenon, stable in 2D gate combinations but not extending to 3D and higher.

Mechanistically, this could arise if:

- Pairwise gates define complementary constraints that mutually stabilize utility regions.
- Triple gates over-constrain the system, causing intersection regions to collapse or become fragmented.
- Non-additive coupling terms decay with gate number, becoming subdominant for $n \geq 3$.

This pattern resembles constraint satisfaction phase transitions in combinatorial optimization, where solution spaces undergo percolation transitions as constraint density increases.

5.4 Synergistic Enhancement vs. Exclusion

A notable asymmetry emerged in P_1-B: all observed pairwise non-additive interactions were synergistic (positive Δ), enhancing utility beyond independent predictions. No exclusion wedges (negative Δ) were detected.

Phase P_2-A detected weak exclusion signals (1.82% volume in Schedule A), but these disappeared under relaxation. This suggests that exclusion mechanisms, if present, are fragile and sensitive to predicate boundaries.

The dominance of synergistic interactions indicates that V-3, V-4, and V-5 represent *complementary* rather than competing constraints. Their simultaneous satisfaction may be mutually reinforcing, analogous to how multiple chemical bonds can stabilize molecular configurations beyond additive expectations.

5.5 Methodological Significance

Beyond the empirical findings, this work demonstrates the effectiveness of preregistration discipline in preventing overinterpretation:

- P_1-C detected a suggestive signal but failed preregistered falsification criteria (F1).
- Rather than claiming discovery, the result was classified as a *constrained-interaction null*.
- Phase P_2-A was designed as an explicit follow-up with authorized predicate relaxation.
- The P_2-A null result definitively resolved the ambiguity without retroactively invalidating P_1 findings.

This cumulative approach—where each phase either validates or bounds prior claims—exemplifies reproducible experimental physics methodology applied to computational systems research.

5.6 Comparison to Additive Baseline

To quantify the departure from additivity in the validated pairwise case, we computed:

$$f_{\text{NA}} = \frac{\#\text{ points with } |\Delta| \geq 0.10}{\#\text{ points in overlap region}} = \frac{126}{126} = 1.00$$

Under an additive model, small residuals $|\Delta| < 0.10$ would arise purely from finite-sample noise (50 seeds). Bootstrap analysis indicates that noise-driven residuals would exceed 0.10 in $< 5\%$ of points. The observed 100% exceeds this by a factor of 20, yielding $p < 10^{-12}$.

For the triple case (P_2-A, Schedule C), the corresponding metric is:

$$f_{\text{NA}}^{(3)} = \frac{3.96\%}{83.3\%} = 0.048$$

This 4.8% non-additive fraction is consistent with noise-driven fluctuations, supporting the interpretation that triple interaction is not a genuine structural feature.

6 Relation to Existing Theories

Many frameworks in complex systems, computation, and physics implicitly assume additive feasibility. Constraints are typically modeled as separable conditions (independent filters), effective potentials (additive energy contributions), or factorizable probability distributions.

Our findings suggest an alternative view: *feasibility itself can generate geometry.* This aligns with perspectives in:

- **Constraint Satisfaction Landscapes:** Phase transitions in random k-SAT and related NP-complete problems exhibit sharp boundaries and clustering phenomena. However, most studies treat different constraint types as independent. Our results suggest that constraint *combinations* may define geometric structures beyond those predicted by individual constraint statistics.

- **Emergent Spacetime Programs:** Approaches such as causal set theory, loop quantum gravity, and tensor network models posit that spacetime geometry emerges from discrete combinatorial structures. While these frameworks focus on causal or entanglement structure, our work suggests that *admissibility geometry* may be a complementary organizational principle.

- **Catastrophe Theory:** Sharp transitions and boundary curvature resemble phenomena in catastrophe theory, where smooth parameter changes induce discontinuous state transitions. However, our boundaries arise from recursive admissibility rather than potential functions, suggesting a distinct mechanism class.

Notably, no appeal is made to optimization, equilibrium, or thermodynamic limits. The observed geometry arises purely from *recursive admissibility constraints*, independent of any minimization principle or energy functional.

7 Limitations and Future Directions

7.1 Triple Interaction Resolution

The primary ambiguity from Phase P_1 (sparse triple overlap in P_1-C) has been definitively resolved by Phase P_2-A. Controlled predicate relaxation across three independent schedules established that triple-gate interaction geometry is not a robust structural feature. This closes the question raised by P_1-C without undermining the validated pairwise findings.

7.2 Limited Gate Coverage

Phase P_1 and P_2-A examined only three gates (V-3, V-4, V-5) from a larger hierarchy. Additional gates (V-6 through V-12) represent distinct mechanism classes including information-theoretic constraints, thermodynamic bounds, and compositional rules.

Future phases will test whether:

- Different pairwise combinations (e.g., V-3 $\times$ V-6, V-4 $\times$ V-7) exhibit comparable non-additive structure
- Certain gate pairs suppress rather than enhance utility (exclusion mechanisms)
- The dimensional constraint ($n = 2$ interaction only) holds across all gate combinations

7.3 Single Seed Block

All Phase P_1 and P_2-A measurements used seeds 196884–196933. While this ensures reproducibility, it does not test generality across seed distributions. Future phases will employ randomized seed selection to validate that admissibility geometry is seed-invariant.

Preliminary analysis suggests that gate boundaries (e.g., $\beta = 0.065$ for V-3) are seed-independent, but interaction magnitudes may vary across seed distributions.

7.4 Binary Utility Definition

The current utility definition is binary (success/failure at threshold $\tau = 0.80$). Continuous utility metrics (e.g., growth rate, stability indices, recursion depth achieved) may reveal finer structure within feasibility boundaries.

For instance, configurations near but not exceeding the utility threshold may exhibit characteristic signatures (pre-critical fluctuations, transient stability) that could inform mechanism design.

7.5 Substrate Specificity

All findings derive from the UNNS recursive substrate with specific gate implementations. Generalization to other recursive systems (cellular automata, Turing machines, neural network architectures) remains an open question.

However, the conceptual framework—admissibility as geometric property of histories rather than configurations—is substrate-independent and may apply broadly.